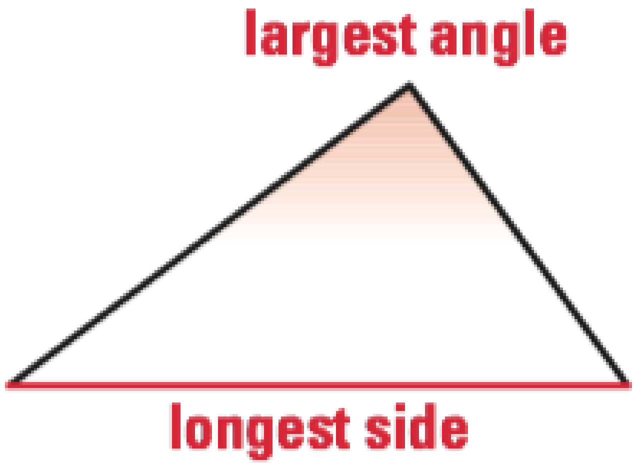
Chapter 5

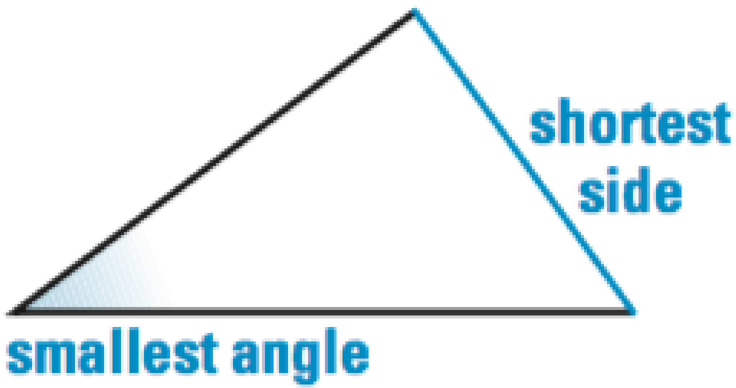
Properties of Triangles

Section 5
Inequalities in One Triangle

GOAL 1: Comparing Measurements of a Triangle

Previously, we discovered a relationship between the positions of the longest and shortest sides of a triangle and the positions of its angles.



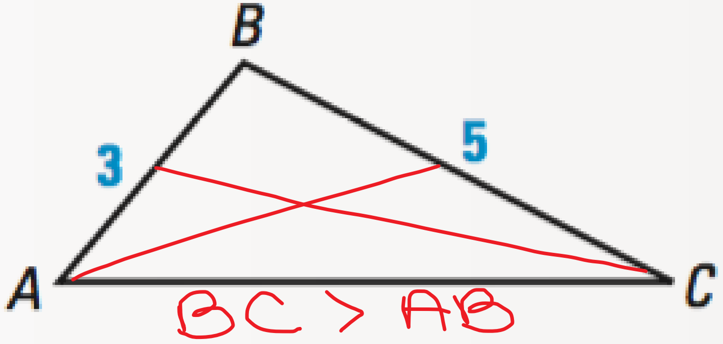


The diagrams illustrate the results stated in the theorems on the next slide.

THEOREMS

THEOREM 5.10

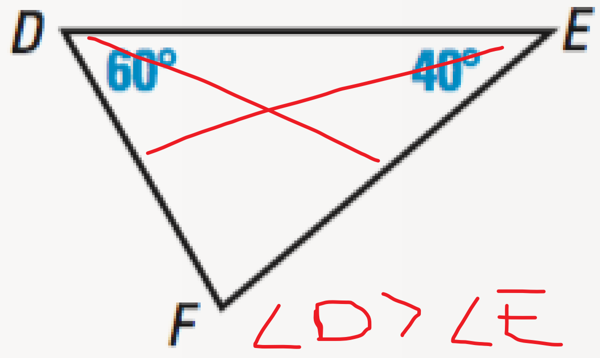
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$m\angle A > m\angle C$

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

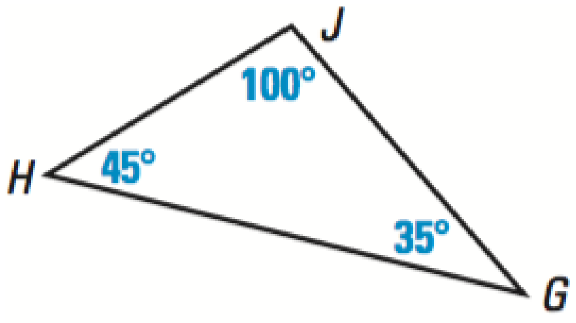


$EF > DF$

Example 1: Writing Measurements in Order from Least to Greatest

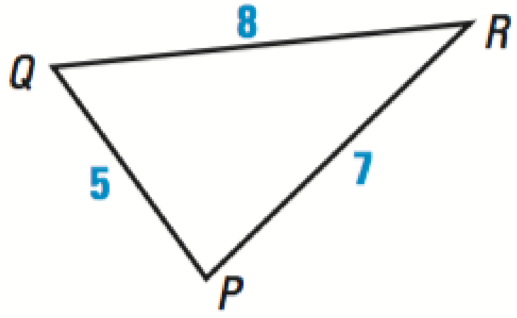
Write the measurements of the triangles in order from least to greatest.

a.



$\angle G$ HJ
$\angle H$ JG
$\angle J$ HG

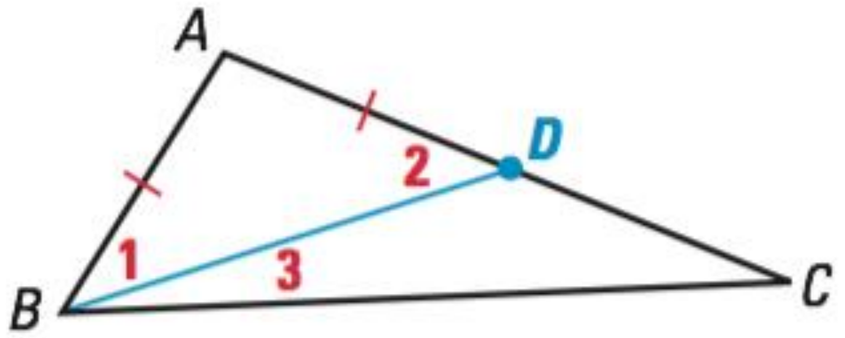
b.



QP $\angle R$
PR $\angle Q$
QR $\angle P$

GIVEN ▶ $AC > AB$

PROVE ▶ $m\angle ABC > m\angle C$



Paragraph Proof Use the Ruler Postulate to locate a point D on $\overline{AC}$ such that $DA = BA$. Then draw the segment $\overline{BD}$. In the isosceles triangle $\triangle ABD$, $\angle 1 \cong \angle 2$. Because $m\angle ABC = m\angle 1 + m\angle 3$, it follows that $m\angle ABC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle ABC > m\angle 2$. Because $m\angle 2 = m\angle 3 + m\angle C$, $m\angle 2 > m\angle C$. Finally, because $m\angle ABC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle ABC > m\angle C$.

The proof of Theorem 5.10 above uses the fact that $\angle 2$ is an exterior angle for $\triangle BDC$, so its measure is the sum of the measures of the two nonadjacent interior angles. Then $m\angle 2$ must be greater than the measure of either nonadjacent interior angle. This result is stated below as Theorem 5.12.

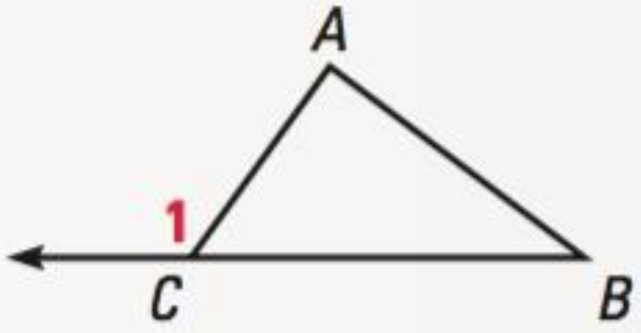
$m\angle A + m\angle B = m\angle 1$

THEOREM

THEOREM 5.12 ***Exterior Angle Inequality***

The measure of an exterior angle of a triangle is greater than the measure of either of the two nonadjacent interior angles.

$m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$



Example 2: Using Theorem 5.10

In the director's chair shown, $\overline{AB} \cong \overline{AC}$ and BC > AB. What can you conclude about the angles in ΔABC?

B/c AB cong. AC → <B cong. <C

B/c BC > AB & AB cong. AC → BC > AC

→ <A > <B, <A > <C

GOAL 2: Using the Triangle Inequality

Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.

Example 3: Constructing a Triangle

Construct a triangle with the given group of side lengths, if possible.

a) 2 cm, 2 cm, 5 cm

(1) (2) (3)

$2+2>5$

$4>5$ F

no $\triangle$

b) 3 cm, 2 cm, 5 cm

(1) (3) (2)

$3+5>2$ ✓

$5+2>3$ ✓

$3+2>5$ F

no $\triangle$

c) 4 cm, 2 cm, 5 cm

(1) (2) (3)

$4+2>5$ ✓

$2+5>4$ ✓

$4+5>2$ ✓

yes $\triangle$

THEOREM

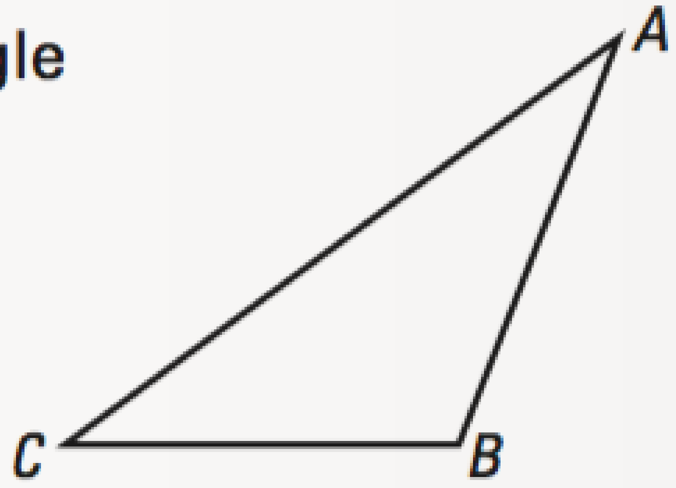
THEOREM 5.13 ***Triangle Inequality***

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$AB + BC > AC \qquad 1 + 2 > 3$$

$$AC + BC > AB \qquad 3 + 2 > 1$$

$$AB + AC > BC \qquad 1 + 3 > 2$$



Example 4: Finding Possible Side Lengths

1 2 3

A triangle has one side of 10 centimeters and another of 14 centimeters. x
Describe the possible lengths of the third side.

$10 + 14 > x$

$\boxed{24 > x}$

$14 + x > 10$

$\cancel{x > -4}$

$10 + x > 14$

$\boxed{x > 4}$

$4 < x < 24$

EXIT SLIP